THC Financial Engineering
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Risk Modeling Bulletin Issue 11

Linear Path Space

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Valuing mortgages involves generating numerous arbitrage-free interest rate scenarios. The process is exceedingly computing intensive. Numerical techniques are used to select these scenarios efficiently. The Feature Article in this issue describes the Linear Path Space approach to valuing mortgages. In the Market Perspective, this method is used to value the hybrid ARMs. The valuation model captures the refinance behavior as well as the default rates of the loans. THC Decisions Feature describes how to perform valuations of hybrid ARMs.

Feature Article: Linear Path Space (LPS) Methodology

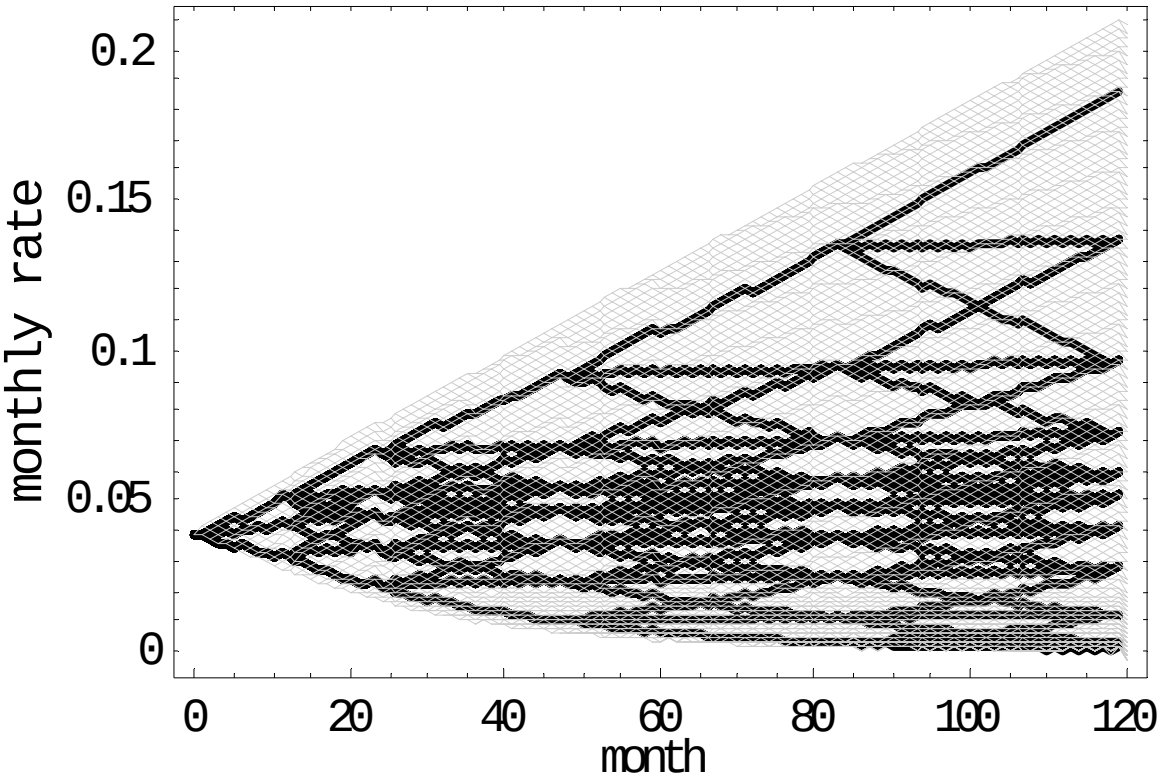
How does a mortgage valuation model capture all the complexities of the mortgagor's behavior? The model does so by simulating an exhaustive set of scenarios and determines the average value of all the possible outcomes. But this approach would require a tremendous amount of computing capacity. Linear Path Space (LPS) is a numerical method that solves this problem. Bulletin 10 presents the Generalized Ho-Lee model, which is an arbitrage-free interest rate binomial lattice model. Consider figure 1 below, the binomial lattice has 120 time steps. There are 2^{120} (= 1.33+E36) paths in this lattice. The LPS approach is to partition this space into a manageable number of equivalent classes of paths, and then determine a "representative path" for each class to which an appropriate weight is assigned. These equivalent classes are determined as follows.

First, we select 5 segments, the 12^{th}, 24^{th}, 48^{th}, 83^{rd}, 119^{th} time steps. We divide the first segment into three gates with equal lengths. These gates are then extended to the second segment and then the 3^{rd}, 4^{th} and 5^{th} segments. The representative paths are the paths that pass through the midpoints of the gates of each segment, and there are now $243(=3^5)$ representative paths. The weight (w_i) is the proportion of the number of paths in a class in which each path has the same meaning with that of the representative path to the total number of paths in the lattice.

To value a mortgage, we now determine only the outcomes of these representative paths and the value of the mortgage is the weighted average of the values (instead of the simple average), based on the weights calculated. In essence, LPS is a structured sampling methodology, first stratifying the binomial path space and then determining the representative paths with their appropriate weights to describe the entire population.

Empirical tests have shown that this method significantly reduces the computation time.

FIGURE 1



Market Perspective: Hybrid ARMs Valuation

Hybrid ARMs are mortgages with a fixed interest rate for an initial period, called the fixed period, after which the rate is adjusted annually based on a certain spread over an agreed upon index rate. Hybrid ARMs valuation depends on the hybrid ARMs prepayment model, interest rate model and the LPS algorithm. The model takes the period cap, initial period cap, and life cap into account. The hybrid ARM considered below is based on the one year LIBOR index.

Consider the following simulation based on June 30, 2006 market data. Figure 2 shows the performance profiles of the 3/1, 5/1 hybrid ARMs and 30yr fixed rate mortgage described in Tables 1, 2 and 3 respectively. Compared to the 30yr FRM, both hybrid ARMs have lower durations and are less negatively convex in the falling rate regimes.

TABLE 1: Details of a 3/1 Hybrid ARM

Start Date	Maturity	Original Coupon(%)	Index	Period Cap(BP)	Initial Period Cap Up/Down(BP)	LifeCap (BP)	Margin (BP)
2006-3-31	2036-3-31	5.63	1-Year LIBOR	200	200/200	1163	225

TABLE 2: Details of a 5/1 Hybrid ARM

Start Date	Maturity	Original Coupon(%)	Index	Period Cap(BP)	Initial Period Cap Up/Down(BP)	LifeCap (BP)	Margin (BP)
2006-3-31	2036-3-31	5.67	1-Year LIBOR	200	500/500	1067	225

TABLE 3: Details of a 30yr Fixed Rate Mortgage Loan

Start Date	Maturity	WAC(%)	OAS(%)
2006-3-31	2036-3-31	6.47	0.65

FIGURE 2

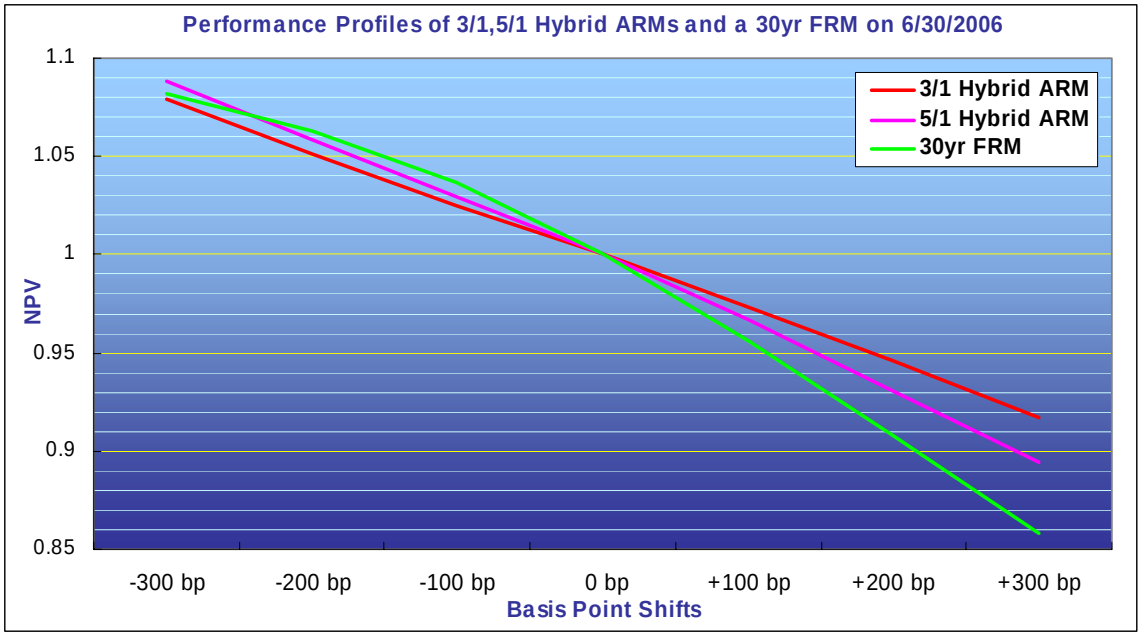
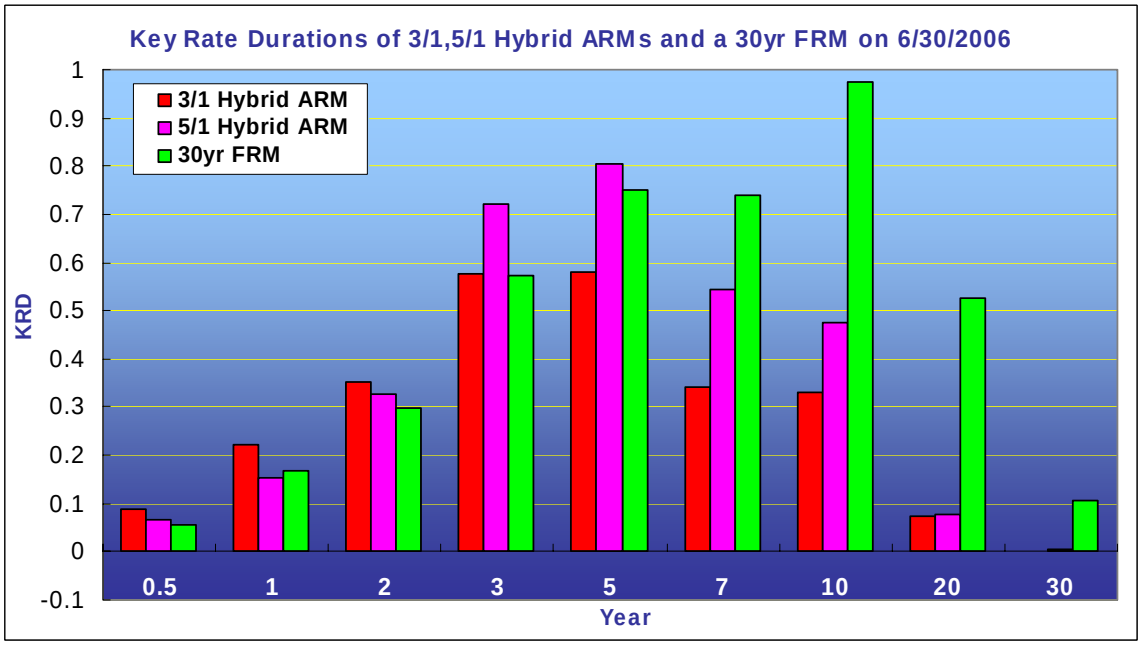


Table 4 and figure 3 depict both key rate durations and effective durations of the hybrid ARMs and the 30yr FRM. The 3/1 ARM has the lowest duration and the 30yr FRM has the highest. For both of the hybrid ARMs, a lower duration can be significantly attributed to the lower key rate durations at 7, 10 and 20 years, which can be seen from figure 3.

TABLE 4: Key Rate Durations of the ARMs and 30yr FRM

	Key Rate Duration									Duration
	0.5	1	2	3	5	7	10	20	30	
3/1 ARM	0.09	0.22	0.35	0.58	0.58	0.34	0.33	0.07	0.00	2.57
5/1 ARM	0.07	0.15	0.33	0.72	0.81	0.55	0.48	0.08	0.01	3.11
30yr FRM	0.06	0.17	0.30	0.57	0.75	0.74	0.97	0.53	0.10	4.03

FIGURE 3



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